



Rayleigh based SPRT: Order Statistics

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Abstract:- Noise is integral in the software failure data. A conversion of data is needed to smooth out the noise. Smoothing enhances quality at first as the size increments and turns out to be more regrettable as the gathering size is vast. Order statistics deals with applications of ordered random variables and functions of these variables. When failures are frequent or inter failure time is less, the use of order statistics is significant. Classical Hypothesis testing needs more time to draw conclusions by collecting volumes of data. But, to decide upon the reliability or unreliability of the developed software very quickly Sequential Analysis of Statistical science could be adopted. The method embraced for this is, Sequential Probability Ratio Test (SPRT), which is designed for continuous monitoring. The likelihood based SPRT proposed by Wald is very general and it can be used for many different probability distributions. The method used to derive the unknown parameters is Maximum Likelihood Estimation (MLE). In this paper, a control mechanism based on Order statistics and Sequential Probability Ratio Test is applied using mean value function of Rayleigh distribution and analysed the results.

Keywords — Rayleigh, Maximum Likelihood Estimation, Order statistics, SPRT, Reliability.

1. INTRODUCTION

In developing and testing new software products, Software reliability assessment is increasingly important. Before releasing the software into the market, the newly developed software is tested extensively to detect errors. New errors may creep into when the detected errors are removed during debugging. The failure costs will be high, if the software with errors is released into the market. In this paper the reliability is assessed by applying SPRT on ordered statistics failure data.

1.1. ORDERED STATISTICS

Let 'X' denote a continuous random variable with probability density function, $f(x)$ and cumulative distribution function, $F(x)$. Let (X_1, X_2, \dots, X_k) denote a random sample of size 'k' drawn on X. The original sample observations may be unordered with respect to magnitude. A conversion is required to produce a corresponding ordered sample. Let $(X_{(1)}, X_{(2)}, \dots, X_{(k)})$

denote the ordered random sample such that $X_{(1)} < X_{(2)} < \dots < X_{(k)}$;

then $X_{(1)}, X_{(2)}, \dots, X_{(k)}$ are collectively known as the order statistics derived from the parent A. The various distributional characteristics can be known from Balakrishnan and Cohen (1991).

The Time Between failure data represent the time laps between every two consecutive failures. The change being applied is, the failure data is made into groups of 4, 5 and then cumulated. On the other hand if a reasonable waiting time for failures is not a serious problem we can group the inter failure time data into non overlapping successive subgroups of size 4 or 5 and add the failures times with needs of groups. For instance if a data of 100 inter failure times are available, we can group them into 20 disjoint subgroups of size 5. The sum totals in each subgroup would represent the time laps between every 5th failures. In the theory of statistics such a subtotal is defined as the 5th order statistics in a sample of size 5.

In general for TBF data of size ‘c’. if ‘r’ is any natural number less than ‘c’ and preferably a factor of ‘c’, we can expediently divide the data into ‘d’ disjoint subgroups ($d = c / r$) and the cumulative total meets subgroup indicate the time between every r^{th} failure. The probability distribution of such a time laps would be better in the r^{th} order statistic in a subgroup of size ‘r’. This would be equal to the r^{th} power of the distribution function of the original variable.

In the present paper, Rayleigh distribution is used to assess the software reliability based on the cumulative Time between Failures (TBF) data which is ordered through a conversion. The parameters of the mean value function with the revised distribution function would determine the constraints involving order statistics, considering $r = 4, 5$. Choice of ‘r’ beyond 5 may create an overly long waiting time for the occurrence of every r^{th} failure (Krishna Mohan et. al, 2011).

1.2. SEQUENTIAL PROBABILITY RATIO TEST

Whenever it requires a decision between two simple hypotheses or a single decision point, Sequential Probability Ratio Test (SPRT) can be applied. SPRT procedure proposed by Wald’s (1947) can be used to classify the software under test into one of two categories i.e reliable or unreliable (Reckase, 1983). If the data collected is sequential, Wald’s procedure is particularly appropriate. Sequential Analysis is different from Classical Hypothesis Testing. In Classical Hypothesis testing, the number of cases tested or collected is fixed at the beginning of the experiment. In this strategy, the analysis is made after gathering the complete data and then the conclusions are drawn. But, in Sequential Analysis every case is analysed directly. The data collected up to that moment is then compared with threshold values, incorporating the new information taken from the freshly collected case. This approach makes one to draw conclusions during the data collection, and ultimate conclusion can be reached at a much earlier stage. Data collection can be terminated after few cases and decisions can be taken quickly. This leads to saving in terms of cost and human life.

In the analysis of software failure data, either TBFs or failure count in a given time interval is dealt with. If it is further assumed that the average number of recorded failures in a given time interval is directly proportional to the length of the interval and the random number of failure occurrences in the interval is explained by a Poisson process. Then it is known that the probability equation of the random process representing the failure occurrences is given by a Homogeneous Poisson Process (HPP) with the expression

$$P[N(t) = n] = \frac{e^{-\lambda t} (\lambda t)^n}{n!}$$

If classical testing strategies are used, the application of SRGM(Software Reliability Growth Model)s may be difficult and reliability predictions can be misleading (Stieber,

1997). However, he observes that statistical methods can be successfully applied to the failure data. He demonstrated his observation by applying the well-known SPRT of Wald for a software failure data to detect unreliable software components and compare the reliability of different software versions. In this paper the popular SRGM – Rayleigh is considered and the principle of Stieber is adopted in detecting unreliable software in order to accept or reject the developed software. The theory proposed by Stieber is presented in Section 2. Extension of this theory to the considered SRGM is presented in Section 3. Maximum Likelihood Estimation (MLE) method is presented in Section 4 to estimate the unknown parameters. Application of the decision rule to detect unreliable software with reference to the SRGM-Rayleigh is given in Section 5.

2. SEQUENTIAL TEST FOR A POISSON PROCESS

A.Wald, developed the SPRT at Columbia University in 1943. A big advantage of sequential tests is that they require fewer observations (time) on the average than fixed sample size tests. SPRTs are widely used for statistical quality control in manufacturing processes. The SPRT for HPP is described below.

Let $\{N(t), t \geq 0\}$ be a HPP process with rate ‘ λ ’.

In this case, $N(t)$ = number of failures up to time ‘t’ and ‘ λ ’ is the failure rate. If the system is put on test and that if we want to estimate its failure rate ‘ λ ’. We cannot expect to estimate ‘ λ ’ precisely. If the data suggest that the failure rate is larger than λ_1 , the system is rejected with high probability and if it is smaller than λ_0 , the system is accepted it with a high probability. As there is some risk to get the wrong answers always with statistical tests, we have to specify two numbers ‘ α ’ and ‘ β ’, where ‘ α ’ is the probability of falsely rejecting the system i.e rejecting the system even if $\lambda \leq \lambda_0$. This is the "producer’s" risk. ‘ β ’ is the probability of falsely accepting the system i.e accepting the system even if $\lambda \leq \lambda_1$. This is the “consumer’s” risk. Wald’s classical SPRT is very sensitive to the choice of relative risk required in the specification of the alternative hypothesis. With the classical SPRT, tests are performed continuously at every time point $t > 0$ as additional data are collected. With specified choices of λ_0 and λ_1 such that $0 < \lambda_0 < \lambda_1$, the probability of finding $N(t)$ failures in the time interval $(0, t)$ with λ_1, λ_0 as the failure rates are respectively given by

$$P_1 = \frac{e^{-\lambda_1 t} [\lambda_1 t]^{N(t)}}{N(t)!} \tag{2.1}$$

$$P_0 = \frac{e^{-\lambda_0 t} [\lambda_0 t]^{N(t)}}{N(t)!} \tag{2.2}$$

The ratio $\frac{P_1}{P_0}$ at any time 't' is considered as a measure of deciding the truth towards λ_0 or λ_1 , given a sequence of time instants say $t_1 < t_2 < t_3 < \dots < t_K$ and the corresponding realizations $N(t_1), N(t_2), \dots, N(t_K)$ of $N(t)$. Simplification of $\frac{P_1}{P_0}$ gives

$$\frac{P_1}{P_0} = \exp(\lambda_0 - \lambda_1)t + \left(\frac{\lambda_1}{\lambda_0}\right)^{N(t)}$$

The decision rule of SPRT is to decide in favor of λ_1 , in favor of λ_0 or to continue by observing the number of failures at a later time than 't' according as $\frac{P_1}{P_0}$ is greater than or equal to a constant say A, less than or equal to a constant say B or in between the constants A and B. That is, we decide the given software product as unreliable, reliable or continue (Satyaprasad, 2007) the test process with one more observation in failure data, according to

$$\frac{P_1}{P_0} \geq A \tag{2.3}$$

$$\frac{P_1}{P_0} \leq B \tag{2.4}$$

$$B < \frac{P_1}{P_0} < A \tag{2.5}$$

The approximate values of the constants A and B are taken as

$$A \cong \frac{1-\beta}{\alpha}, \quad B \cong \frac{\beta}{1-\alpha}$$

Where ' α ' and ' β ' are the risk probabilities as defined earlier. A good test is one that makes the α and β errors as small as possible. The common procedure is to fix the β error and then choose a critical region to minimize the error or maximize the power i.e $1-\beta$ of the test. A simplified version of the above decision processes is to reject the system as unreliable if $N(t)$ falls for the first time above the line

$$N_U(t) = a.t + b_2 \tag{2.6}$$

if $N(t)$ falls for the first time below the line, accept the system to be reliable

$$N_L(t) = a.t - b_1 \tag{2.7}$$

If the random graph of $[t, N(t)]$ is between the two linear boundaries given by equations (2.6) and (2.7), continue the test with one more observation on $(t, N(t))$. Where,

$$a = \frac{\lambda_1 - \lambda_0}{\log\left(\frac{\lambda_1}{\lambda_0}\right)} \tag{2.8}$$

$$b_1 = \frac{\log\left[\frac{1-\alpha}{\beta}\right]}{\log\left(\frac{\lambda_1}{\lambda_0}\right)} \tag{2.9}$$

$$b_2 = \frac{\log\left[\frac{1-\beta}{\alpha}\right]}{\log\left(\frac{\lambda_1}{\lambda_0}\right)} \tag{2.10}$$

The parameters α, β, λ_0 , and λ_1 can be chosen in several ways. One way suggested by Stieber is $\lambda_0 = \frac{\lambda \cdot \log(q)}{q-1}$,

$$\lambda_1 = q \frac{\lambda \cdot \log(q)}{q-1} \quad \text{where } q = \frac{\lambda_1}{\lambda_0}$$

The slope of $N_U(t)$ and $N_L(t)$ equals λ , If λ_0 and λ_1 are chosen in this way. The other two ways of choosing λ_0 and λ_1 are from past projects and from part of the data to compare the reliability of different functional areas.

3. SEQUENTIAL TEST FOR SOFTWARE RELIABILITY GROWTH MODELS

In Section 2, it is known that the expected value of $N(t) = \lambda t$ called the average number of failures experienced in time 't' for a Poisson process. This is also called the mean value function of the Poisson process. On the other hand, if we consider a Poisson process with a general function $m(t)$ as its mean value function the probability equation of such a process is

$$P[N(t) = Y] = \frac{[m(t)]^y}{y!} \cdot e^{-m(t)}, \quad y = 0, 1, 2, \dots$$

Various NHPPs are obtained depending on the forms of $m(t)$. For a two parameter Rayleigh model (Weibull, 1951),

the mean value function is given as $m(t) = a(1 - e^{-(bt)^2})$

where $a > 0, b > 0$

It may be written as

$$P_1 = \frac{e^{-m_1(t)} \cdot [m_1(t)]^{N(t)}}{N(t)!}$$

$$P_0 = \frac{e^{-m_0(t)} \cdot [m_0(t)]^{N(t)}}{N(t)!}$$

Let P_0, P_1 be values of the NHPP at two specifications of 'b' say b_0, b_1 , where $(b_0 < b_1)$. Where, $m_1(t), m_0(t)$ are the mean value function at specified sets of its parameters indicating reliable and unreliable software respectively. It can be shown that for our model $m(t)$ at b_1 is greater than that at b_0 . Symbolically $m_0(t) < m_1(t)$. Then the SPRT procedure is as follows (Krishna Mohan and Satya Prasad, 2011):

If $\frac{P_1}{P_0} \leq B$, Accept the system as reliable.

$$\text{i.e., } \frac{e^{-m_1(t)} \cdot [m_1(t)]^{N(t)}}{e^{-m_0(t)} \cdot [m_0(t)]^{N(t)}} \leq B$$

$$\text{i.e., } N(t) \leq \frac{\log\left(\frac{\beta}{1-\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \quad (3.1)$$

If $\frac{P_1}{P_0} \geq A$, reject the system as unreliable.

$$\text{i.e., } N(t) \geq \frac{\log\left(\frac{1-\beta}{\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \quad (3.2)$$

Otherwise, Continue the test procedure as long as

$$\frac{\log\left(\frac{\beta}{1-\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} < N(t) < \frac{\log\left(\frac{1-\beta}{\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \quad (3.3)$$

Substituting the appropriate expressions of the respective mean value function $-m(t)$ of Rayleigh we get the respective decision rules and are given in following lines
Acceptance region:

$$N(t) \leq \frac{\log\left(\frac{\beta}{1-\alpha}\right) + a\left(e^{-(b_1t)^2} - e^{-(b_0t)^2}\right)}{\log\left(\frac{1 - e^{-(b_1t)^2}}{1 - e^{-(b_0t)^2}}\right)} \quad (3.4)$$

Rejection region:

$$N(t) \geq \frac{\log\left(\frac{1-\beta}{\alpha}\right) + a\left(e^{-(b_0t)^2} - e^{-(b_1t)^2}\right)}{\log\left(\frac{1 - e^{-(b_1t)^2}}{1 - e^{-(b_0t)^2}}\right)} \quad (3.5)$$

Continuation region:

$$\frac{\log\left(\frac{\beta}{1-\alpha}\right) + a\left(e^{-(b_0t)^2} - e^{-(b_1t)^2}\right)}{\log\left(\frac{1 - e^{-(b_1t)^2}}{1 - e^{-(b_0t)^2}}\right)} < N(t) < \frac{\log\left(\frac{1-\beta}{\alpha}\right) + a\left(e^{-(b_0t)^2} - e^{-(b_1t)^2}\right)}{\log\left(\frac{1 - e^{-(b_1t)^2}}{1 - e^{-(b_0t)^2}}\right)} \quad (3.6)$$

The decision rules are exclusively based on the strength of the sequential procedure (α, β) and the values of the respective mean value functions namely, $m_0(t), m_1(t)$. The decision rules become decision lines as defined by Stieber, If the mean value function is linear in 't' passing through origin, i.e $m(t) = \lambda t$. The application of these results for software failure data of ordered statistics is presented with analysis in Section 5.

4. MAXIMUM LIKELIHOOD ESTIMATION

Based on the failure data given in Lyu(1996), a transformation is applied to make them to an ordered statistics data by grouping. ' \hat{a} ' and ' \hat{b} ' are MLEs of parameters 'a' and 'b' and the values can be computed using Newton Raphson iterative method.

Mathematical derivation of parameter estimation

The $m(t)$ of a Rayleigh distribution has the form: $m(t) = a[1 - e^{-(bt)^2}]$. For r^{th} order statistics, the $m(t)$ is expressed as $m(t)^r = \left(a[1 - e^{-(bt)^2}]\right)^r$. The failure intensity function is given as: $\lambda^r(t) = 2.a^r . b^2 . r.t(1 - e^{-(bt)^2})^{r-1} . e^{-(bt)^2}$.

To get the estimates of 'a' and 'b', for a sample of n units, the likelihood function must be obtained first.

$$\text{The Likelihood function, } L = e^{-m^r(t_n)} \prod_{i=1}^n \lambda^r(t_i). \quad (4.1)$$

Taking the natural logarithm on both sides, The Log Likelihood function is given as:

$$\log L = \sum_{i=1}^n \log\left(2.a^r . b^2 . r.t(1 - e^{-(bt)^2})^{r-1} . e^{-(bt)^2}\right) - \left(a[1 - e^{-(bt)^2}]\right)^r \quad (4.2)$$

The parameter ' \hat{a} ' is estimated by taking the partial derivative w.r.t 'a' and equating to '0'.

$$a^r = \frac{n}{\left[1 - e^{-(bt)^2}\right]^r} \quad (4.3)$$

The parameter ' \hat{b} ' is estimated by iterative Newton Raphson Method i.e $b_{n+1} = b_n - \frac{f(b_n)}{f'(b_n)}$, which is substituted in finding ' \hat{a} '. Where $f(b)$ & $f'(b)$ are expressed as follows.

Taking the Partial derivative w.r.t ' b ' and equating to '0'.

$$f(b) = \frac{2n}{b} - 2b \sum_{i=1}^n t_i^2 + 2(r-1) \sum_{i=1}^n \frac{(t_i)^2 b e^{-(bt_i)^2}}{(1 - e^{-(bt_i)^2})} - n. \quad (4.4)$$

Again by partially differentiating w.r.t ' b ' and equating to 0.

$$f'(b) = -\frac{2n}{b^2} - 2 \sum_{i=1}^n t_i^2 + 2(r-1) \sum_{i=1}^n \frac{(t_i)^2 \left[e^{-(bt_i)^2} (1 - e^{-(bt_i)^2}) - 2bt_i^2 e^{-(bt_i)^2} \right]}{(1 - e^{-(bt_i)^2})^2} \quad (4.5)$$

5. SPRT ANALYSIS OF DATA SETS

In this section, the developed SPRT methodology is shown for a software failure data which is of time domain in 4th and 5th order. In this section the decision rules based on the considered mean value function for FIVE different data sets borrowed from Lyu(1996) are evaluated. Based on the estimates of the parameter ' b ' in each mean value function, we have chosen the specifications of $b_0 = b - \delta$, $b_1 = b + \delta$ equidistant on either side of estimate of b obtained through a data set to apply SPRT such that $b_0 < b < b_1$. Assuming the value of $\delta = 0.00001$, the choices are given in the following table.

5.1. ESTIMATED PARAMETERS

The estimated parameters for SYS1, SYS2, SYS3, CSR2 & CSR3 datasets are given in Table 5.1.1. The estimated values of ' a ' and ' b ' and their control limits for both 4th-order and 5th-order statistics are as follows.

Table 5.1.1: Estimates of a, b & Specifications of b_0, b_1 for 4 & 5 order

Data Set	Order	Estimated Parameters			
		a	b	b_0	b_1
1	4	2.414736	0.000049	0.000039	0.000059
	5	1.933182	0.000058	0.000048	0.000068
2	4	2.378414	0.000054	0.000044	0.000064
	5	1.903654	0.000068	0.000058	0.000078
3	4	2.258104	0.000239	0.000229	0.000249
	5	1.820564	0.000297	0.000287	0.000307
4	4	2.140702	0.000036	0.000026	0.000046

	5	1.762340	0.000041	0.000031	0.000051
5	4	2.672364	0.000213	0.000203	0.000223
	5	2.101635	0.000228	0.000218	0.000238

Using the selected b_0, b_1 and subsequently the $m_0(t), m_1(t)$ for the model, we calculated the decision rules given by Equations 3.4 and 3.5, sequentially at each ' t ' of the data sets taking the strength (α, β) as (0.05, 0.2).

5.2 ANALYSIS

Table 5.2.1: 4th order SPRT analysis for 5 data sets

Data Set	T	N(t)	Acceptance region (\leq)	Rejection Region (\geq)	Decision
1	227	1	-1.881518	3.348823	Rejection
	444	2	-1.880362	3.349078	
	759	3	-1.877358	3.349737	
	1056	4	-1.873097	3.350665	
2	1557	1	-2.055537	3.703353	Rejection
	1639	2	-2.053000	3.703719	
	1973	3	-2.041376	3.705356	
	2183	4	-2.033026	3.706496	
3	112	1	-9.299533	16.552234	Rejection
	293.5	2	-9.270615	16.526925	
	473.5	3	-9.216835	16.479392	
	630.5	4	-9.150165	16.419619	
	793.5	5	-9.062334	16.339419	
	955.5	6	-8.957317	16.241315	
	1171.5	7	-8.791998	16.081868	
	1323.5	8	-8.659985	15.950016	
	1443.5	9	-8.547577	15.834477	
	1810.5	10	-8.166239	15.419101	
	1924.5	11	-8.038401	15.271389	
	2446.5	12	-7.417203	14.488189	
	3304.5	13	-6.362303	12.880401	
4	1576	1	-1.356674	2.432645	Rejection
	4149	2	-1.305847	2.448472	
	5827	3	-1.250988	2.464053	
5	89	1	-8.288573	14.751492	Rejection

1	193	2	-8.279522	14.745233	Rejection
	269	3	-8.268712	14.737732	
	354	4	-8.252460	14.726405	
	482	5	-8.219805	14.703463	
	796	6	-8.099546	14.616821	
	1257	7	-7.829095	14.409082	
	1519	8	-7.633101	14.246717	
	1566	9	-7.595195	14.214110	
	1873	10	-7.330247	13.974727	
	1940	11	-7.268939	13.916366	
	2155	12	-7.065510	13.714277	
	2289	13	-6.934488	13.576963	
	2338	14	-6.885931	13.524593	

4	2610	1	-1.536735	2.788487	Rejection
	4436	2	-1.485538	2.794235	
	8163	3	-1.319217	2.799808	
5	93	1	-8.872789	15.791013	Rejection
	243	2	-8.855709	15.775834	
	345	3	-8.835466	15.757763	
	482	4	-8.797469	15.723599	
	801	5	-8.662801	15.599932	
	1496	6	-8.173857	15.115316	
	1532	7	-8.142444	15.082177	
	1873	8	-7.822166	14.729605	
	1976	9	-7.718504	14.609565	
	2236	10	-7.446035	14.279523	
	2325	11	-7.349925	14.157918	
	2580	12	-7.069075	13.786376	
	2936	13	-6.669993	13.214887	
	3221	14	-6.350766	12.718916	

Table 5.2.2: 5th order SPRT analysis for 5 data sets

Data Set	T	N(t)	Acceptance region (\leq)	Rejection Region (\geq)	Decision
1	342	1	-2.235554	3.980074	Rejection
	571	2	-2.233433	3.980032	
	968	3	-2.227252	3.979897	
	1986	4	-2.197229	3.978953	
2	1579	1	-2.593238	4.673958	Rejection
	1738	2	-2.585648	4.672774	
	2030	3	-2.569936	4.670226	
	2714	4	-2.524544	4.662116	
	3491	5	-2.459633	4.648494	
3	112.5	1	-11.556372	20.569239	Rejection
	358.5	2	-11.479284	20.482808	
	615.5	3	-11.315980	20.296614	
	793.5	4	-11.156667	20.110857	
	1109.5	5	-10.791322	19.669132	
	1246.5	6	-10.604375	19.434406	
	1438.5	7	-10.318161	19.063312	
	1810.5	8	-9.701337	18.213682	
	1939.5	9	-9.473665	17.882344	
	2759.5	10	-7.968047	15.441992	
	3999.5	11	-5.862163	11.364030	
	4493.5	12	-5.144877	9.863770	

From the above table it is observed that a decision of either to accept, reject or continue the system is reached well in advance rather than to wait for the last time instant of the data. For further analysis, assuming the value of $\delta = 0.00005$, the Data sets are rejected at 1st, 1st, 4th, 1st and 3rd for 4th order and at 1st, 1st, 5th, 1st, and 4th for 5th order.

6. CONCLUSION

The table 5.2.1 of 4th order data and Table 5.2.2 of 5th order data as exemplified for 5 Data Sets shows that Rayleigh model is performing well in arriving at a decision. The procedure applied on the model has given a decision of rejection for all the failure data sets under consideration. Data Set#1, #2, #3, #4 and #5 are rejected at 4th, 4th, 13th, 3rd and 14th instant of time for 4th order and are rejected at 4th, 5th, 12th, 3rd, and 14th instant of time for 5th order. Therefore, by applying SPRT on data sets it can be concluded that we can come to an early conclusion of reliable or unreliable software.

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