

# Implementation of a Fast Sign Detection Algorithm for the RNS Moduli Set $\{2^{N+1} - 1, 2^N - 1, 2^N\}$ , $N = 16, 64$

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**Abstract:** - In this dissertation a fast sign detection algorithm for the residue number system (RNS) moduli set  $\{2^{N+1} - 1, 2^{N-1}, 2^N\}$ ,  $N = 16, 64$  is illustrated. This algorithm allows parallel implementation and completes the implementation with simply modulo  $2n$  additions. This unit can be implemented using one carry save adder, one comparator, one carry generation unit and post processing unit. The results of this algorithm confirm that the area and delay are reduced compared to mixed radix conversion and Chinese remainder theorems. This algorithm is implemented using verilog language tool in Xilinx 13.2 version.

**Keywords:** residue number system (RNS), restricted moduli set, sign detection.

## I. Introduction

The RNS is a unique, non-weighted, carry free number system that provides parallel, high speed and fault tolerant arithmetic operations. The sign information is concealed in each residue digit in an RNS, so sign detection in an RNS is more difficult than that in the weighted number system, where the sign bit is the most significant bit (MSB). Sign detection plays an important role in magnitude comparisons, overflow detection, and in branching operations.

There are quite a lot of researches for a sign detection in RNS moduli. A general theorem is derived by establishing necessary conditions for sign detection [1]. There are two categories based on algorithms. They are ROM based algorithms and specialized algorithms. In the ROM based algorithm one method detects sign for a preferred class of RNS is carried out as a sum modulo 2 of digits in the associated mixed radix system (MRS) [2], in [3] another method a sign detection procedure based on fractional illustration is proposed to lessen the sum modulo  $M$  in the conversion formula to a sum modulo 2. In the specialized algorithms one [4] detects the sign based on the new Chinese remainder theorem (CRT) II, the modulo operations are enclosed by size  $\sqrt{M}$ , another one [5] uses the  $n$ th mixed radix conver-

sion (MRC) for sign detection. A [6] combinational logic is implemented based on  $\{2^{n-1}, 2^n, 2^{n+1}\}$  for sign detection in RNS, and it is not extended for other moduli sets. Recently the moduli set  $\{2^{N+1} - 1, 2^{N-1}, 2^N\}$  only having  $2n$  and  $2n-1$  terms used for sign detection because of its efficiency for modular operations (such as addition, subtraction and multiplication), and reverse conversion [7].

In this phenomenon, a sign detection for the moduli set  $\{2^{N+1} - 1, 2^{N-1}, 2^N\}$  is obtainable. This unit consists of a carry save adder (CSA), a comparator and a carry generation and post processing unit, and it only requires addition of the modulo  $2n$ . This proposed method achieved better efficiency compared to the algorithms based on ROM technology [2], [3] and specialized algorithms [4], [5].

## II. PROPOSED SIGN DETECTION ALGORITHM

A standard RNS is completely distinct for positive integers in the range  $[0, M)$ . An implicit signed number system may be considered to be split into a positive half and negative half of the range to hold negative integers. The dynamic range  $M$  of the moduli set  $\{m_1, m_2, \dots, m_{N-1}, m_N = 2^n\}$  is even. An implicit symbol of the

actual result Y can be obtained in its range  $[-M/2, M/2-1)$ , after conversion from residue number to the weighted number ensuing the noninteger X in the interval  $[0, M/2)$  as follows:

$$Y = \begin{cases} X, & \text{if } 0 \leq X < M/2 \\ X - m, & \text{if } M/2 \leq X < M. \end{cases}$$

The mixed radix CRT obtainable in [8] is as follows.

For  $\{m_1, m_2, \dots, m_N\}$ , the magnitude of a residue number  $X=(x_1, x_2, \dots, x_N)$  is as follows:

$$X = \sum_{j=1}^{N-2} \left( \alpha_{j+1} \prod_{i=1}^{j+1} m_i \right) + \alpha_1 m_1 + \alpha_0$$

where  $\alpha_{j+1} = \lfloor \sum_{i=1}^{j+2} \gamma_i x_i / \prod_{i=1}^{j+1} m_i \rfloor \bmod m_{j+2}$ ,  $\alpha_1 = \lfloor \gamma_1 x_1 + \gamma_2 x_2 \rfloor \bmod m_2$ ,  $\alpha_0 = x_1$ ,  $N > 1$ ,  $\gamma_1 = (N_1 \lfloor N_1^{-1} \bmod m_1 - 1 \rfloor) / m_1$ ,  $\gamma_i = M \lfloor N_i^{-1} \bmod m_i / m_1 m_i \rfloor$ ,  $M = m_1 m_2 \cdot \dots \cdot m_N$ ,  $N_i = M / m_i$ , and the multiplicative inverse  $\lfloor N_i^{-1} \bmod m_i \rfloor$  is defined by  $\lfloor N_i^{-1} \bmod m_i \rfloor N_i \bmod m_i = 1$ , for  $i = 1, 2, 3, \dots, N$ . The floor function is denoted by  $\lfloor \cdot \rfloor$

It converts residue numbers to weighted numbers and it requires only modulo  $m_i$  operations. The calculation procedure for each mixed radix  $\alpha_j$  is independent of others, so the mixed radix coefficients can be computed in a fully parallel method. From this we can say that

For the moduli set  $\{m_1, m_2, \dots, m_{N-1}, m_N = 2^n\}$ , the value of  $\alpha_{N-1}$  is equivalent to  $2n-1$  when the integer X is  $M/2$ .

i.e  $\alpha_{N-1}(M/2) = 2^{n-1}$ .

In the moduli set  $\{m_1, m_2, \dots, m_{N-1}, m_N = 2^n\}$ , for a residue representative number  $(x_1, x_2, \dots, x_N)$ ,  $\alpha_{N-1}$  is

$$\alpha_{N-1} = \left\lfloor \frac{\gamma_1 x_1 + \gamma_2 x_2 + \dots + \gamma_N x_N}{m_2 m_3 \cdot \dots \cdot m_{N-1}} \right\rfloor \bmod 2^n$$

Then the projected sign detection function is

Based on this the sign output is the MSB of  $\alpha_{N-1}$ . Therefore, this needs only one carry generation circuit to get nth bit of  $\alpha_{N-1}$ .

### III. SIGN DETECTION FOR THE MODULI SET $\{2^{N+1}-1, 2^N-1, 2^N\}$

A high efficiency sign detection entity for the moduli set  $\{2^{N+1}-1, 2^N-1, 2^N\}$  is presented here. Based

on the projected algorithm the sign detection unit is concurrent and suitable for VLSI implementation.

The sign detection of  $X = (x_1, x_2, x_3)$  for the moduli set  $\{2^{N+1}-1, 2^N-1, 2^N\}$  is

$$\text{Sgn}(x_1, x_2, x_3) = \text{MSB} (\lfloor \frac{-2x_1 + x_2 + x_3 + (x_2 - x_1)}{2^n - 1} \rfloor \bmod 2^n).$$

In binary illustration, the words  $x_1, x_2, x_3$  are  $n+1, n, n$  bit respectively. We symbolize  $x_{1,n}$  as the  $(n+1)$ th bit of  $x_1$ , and denote  $x'_1$  as the least  $n$  bits of  $x_1$ . Because the word  $x_1$  is with one more bit than  $x_2$  and  $x_3$ .

Take

$$W = 1 - \bar{W} = 1 + \left\lfloor \frac{x_2 - x'_1 - x_{1,n}}{2^n - 1} \right\rfloor = \begin{cases} 0, & \text{if } x_{1,n} = 0 \text{ and } x_2 < x'_1, \text{ or } x_{1,n} = 1 \text{ and } x_2 \leq x'_1 \\ 1, & \text{if } x_{1,n} = 0 \text{ and } x_2 \geq x'_1, \text{ or } x_{1,n} = 1 \text{ and } x_2 > x'_1 \end{cases}$$

We denote,  $x''_1$  as a  $n$ -bit digit that equals  $2x_{1,n-2:0} + x_{1,n}$ , which is concatenated by the least  $n-1$  bits of  $x_1$  and  $x_{1,n}$ . From this we can engrave that

$$\begin{aligned} \alpha_2 &= \lfloor -2x_1 + x_2 + x_3 - x_{1,n} - \bar{w} \rfloor \bmod 2^n \\ &= \lfloor -(2x_{1,n-2:0} + x_{1,n}) + x_2 + x_3 - \bar{w} \rfloor \bmod 2^n \\ &= \lfloor 2^n - 1 - x''_1 + x_2 + x_3 + 1 - \bar{w} \rfloor \bmod 2^n \\ &= \lfloor \bar{x}''_1 + x_2 + x_3 + w \rfloor \bmod 2^n \end{aligned}$$

From the above equation we find MSB of  $\alpha_2$  by the hardware realization using the figure (1). The circuit contains CSA [9], carry generation [10], comparator and post processing units.

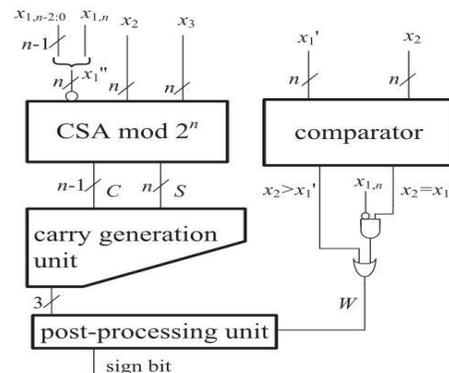


Fig 1: sign detection unit for the RNS moduli set  $\{2^{N+1}-1, 2^N-1, 2^N\}$ .

The CSA mod  $2^n$  is used to execute the sum of  $x^1+x^2+x^3$  and the outputs obtained from this are two vectors sum (S) and carry (C) of n bit. Using carry save addition, the delay can be reduced further still. The thought is to take three inputs that we want to add together,  $x^1 + x^2+x^3$ , and convert it into 2 outputs  $c + s$  such that  $x^1 + x^2+x^3 = c + s$ .

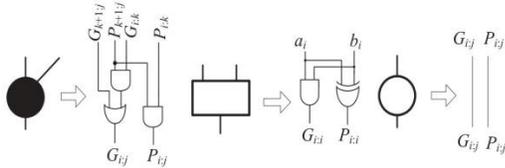


Fig 2: blocks used in carry generation unit and comparator unit.

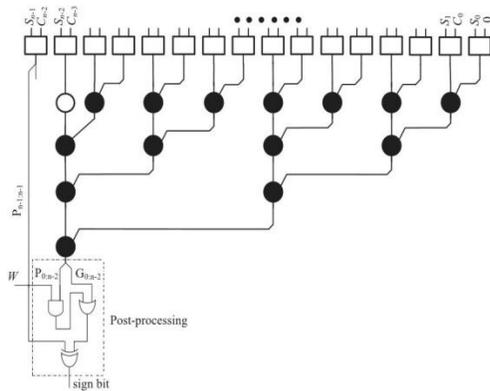


Fig 3: carry generation unit and post processing unit for  $n = 16, 64$ .

The carry generation unit and the post processing unit are used to generate nth bit of  $\alpha_2 = C+S+W$ .

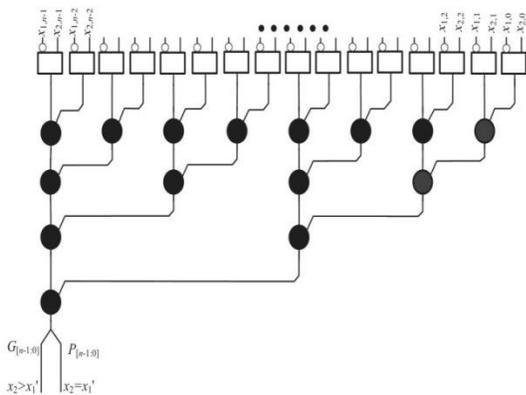


Fig 4: comparator unit for  $n = 16, 64$ .

The comparator unit is used to set up the comparison

of  $x^2 > x^1$  and  $x^2 = x^1$ . In the comparator unit a parallel implementation of the least significant bit first approach comparison algorithm [11] is used. This comparator unit and a carry generation unit are almost same apart from the addition with one input vector being set is ones complement.

#### IV. PERFORMANCE ESTIMATION

The act of the proposed sign detection unit of the moduli set  $\{2^{N+1}-1, 2^N-1, 2^N\}$  is evaluated. The sign detection unit is compared with two units extended by two Best sign detection algorithms to show the high efficiency of the new sign detection algorithm.

The outputs for the sign detection of the RNS moduli set  $\{2^{N+1}-1, 2^N-1, 2^N\}$  is as follows:

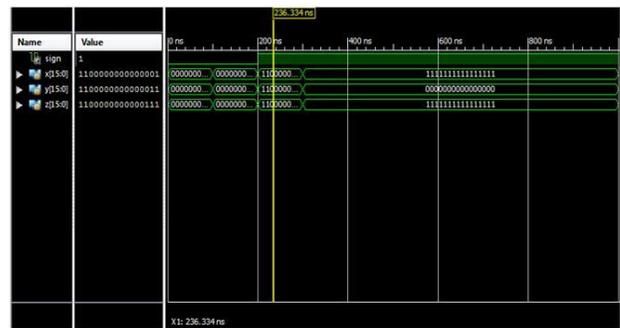


Fig 5. Output for the RNS moduli set  $\{2^{N+1}-1, 2^N-1, 2^N\}$  where  $N=16$ .

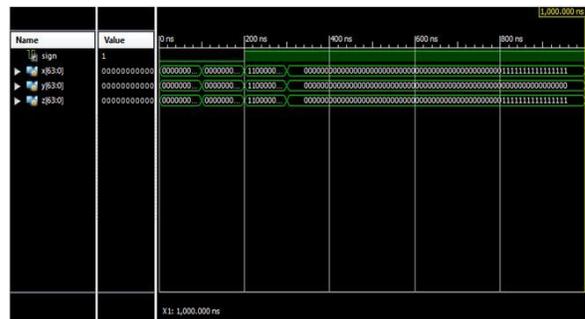


Fig 6. Output for the RNS moduli set  $\{2^{N+1}-1, 2^N-1, 2^N\}$  where  $N=64$ .

From the result we can say that for the inputs  $x=000000000001111$ ,  $y=000000000001111$ ,  $z=000000000001111$ , the sign bit is 0, and for the inputs  $x= 0001111111111111$ ,  $y=1111111111111111$ ,  $z=1111111111111111$ , the sign bit is 1. The results with  $N=64$  bit also verified.

The comparison is done by comparing the proposed

method with the two best existing methods. They are figured below:

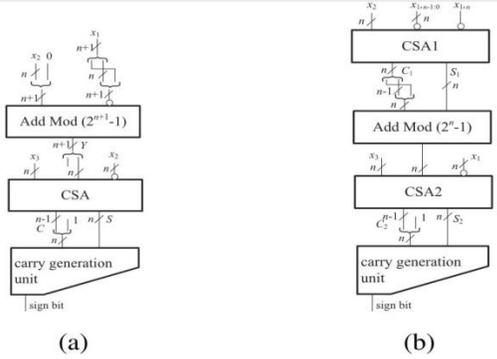


Fig 7. (a) Sign detection unit for the RNS moduli set  $\{2^{n+1}-1, 2^n-1, 2^n\}$  based on [4].  
 (b) Sign detection unit for the RNS moduli set  $\{2^{n+1}-1, 2^n-1, 2^n\}$  based on [5].

The experimental results shows that the area used for the proposed algorithm is compared to the specialized algorithms [4] and [5].

Device Utilization Summary				
Logic Utilization	Used	Available	Utilization	Note(s)
Number of 4 input LUTs	84	7,168	1%	
<b>Logic Distribution</b>				
Number of occupied Slices	46	3,584	1%	
Number of Slices containing only related logic	46	46	100%	
Number of Slices containing unrelated logic	0	46	0%	
<b>Total Number of 4 input LUTs</b>	<b>84</b>	<b>7,168</b>	<b>1%</b>	
Number of bonded IOBs	50	141	35%	
<b>Total equivalent gate count for design</b>	<b>552</b>			
Additional JTAG gate count for IOBs	2,400			

Fig 8: devise utilization summary for the specialized algorithm [4].

In this algorithm the blocks used are one add mod ( $2^{n+1}-1$ ), one CSA, carry generation unit. The total number of gates used for this algorithm is 552, the delay occurred is 22.450ns having levels of logic 14.

Device Utilization Summary				
Logic Utilization	Used	Available	Utilization	Note(s)
Number of 4 input LUTs	47	7,168	1%	
<b>Logic Distribution</b>				
Number of occupied Slices	27	3,584	1%	
Number of Slices containing only related logic	27	27	100%	
Number of Slices containing unrelated logic	0	27	0%	
<b>Total Number of 4 input LUTs</b>	<b>47</b>	<b>7,168</b>	<b>1%</b>	
Number of bonded IOBs	50	141	35%	
<b>Total equivalent gate count for design</b>	<b>297</b>			
Additional JTAG gate count for IOBs	2,400			

Fig 9: devise utilization summary for the specialized algorithm [5].

In this algorithm the blocks used are one add mod ( $2^{n+1}-1$ ), two CSA, carry generation unit. The total number of gates used for this algorithm is 297, the delay occurred is 19.836ns having levels of logic 12.

Device Utilization Summary				
Logic Utilization	Used	Available	Utilization	Note(s)
Number of 4 input LUTs	14	7,168	1%	
<b>Logic Distribution</b>				
Number of occupied Slices	10	3,584	1%	
Number of Slices containing only related logic	10	10	100%	
Number of Slices containing unrelated logic	0	10	0%	
<b>Total Number of 4 input LUTs</b>	<b>14</b>	<b>7,168</b>	<b>1%</b>	
Number of bonded IOBs	36	141	25%	
<b>Total equivalent gate count for design</b>	<b>87</b>			
Additional JTAG gate count for IOBs	1,728			

Fig 10: devise utilization summary for the proposed algorithm.

In this algorithm the blocks used are one CSA, one comparator unit, one carry generation post processing unit and one. The total number of gates used for this algorithm is 87; the delay occurred is 16.016ns having levels of logic 8.

**V. CONCLUSION**

A fast sign detection algorithm for the restricted moduli set  $\{2^{n+1}-1, 2^n-1, 2^n\}$  is proposed with the modulo  $2n$ . The proposed algorithm contains only modulo  $2n$  additions and allows for parallel realization. The proposed algorithm is the first proposed for the moduli set  $\{2^{n+1}-1, 2^n-1, 2^n\}$ . The experimental results show that the proposed circuit achieves considerable improvements in terms of area and delay.

**VI. REFERENCES**

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