Fast Singular value decomposition based image compression using butterfly particle swarm optimization technique (SVD-BPSO)

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Abstract: - Image compression is an important research area in an image processing system. Due to the compression of data rates, this finds crucial in applications of information security for the fast transmission. Singular Value Decomposition (SVD) is a compression technique which performs compression by using a smaller rank to approximate the original matrix of an image. SVD offers good PSNR values with low compression ratios. Compression using SVD for different singular values (Sv) with an acceptable PSNR increases the encoding time (ET). To minimize the encoding time, in this paper a fast compression technique SVD-BPSO is proposed using singular value decomposition and butterfly particle swarm optimization (BPSO). Application of the concept of BPSO towards singular value decomposition reduces the encoding time and improves the transmission speed. The performance of the proposed SVD-BPSO compression method is compared with SVD without optimization technique. The simulation results showed that the method achieves good PSNR with the minimum encoding time.

Keywords: Image Compression, Singular Value Decomposition (SVD), Butterfly Particle Swarm Optimization (BPSO), Encoding.

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1. Introduction

Digital images are enormously used in our daily usages such as telemedicine, astronomy, remote sensing and e-commerce. Hence sharing and storage of digital images are growing rapidly. Large image dataset requires more storage. To minimize the storage requirement, the amount of data used to represent these images has to be reduced. Image compression is the process of reducing the number of bits used to represent an image by removing the redundant information. Image redundancy comes under three categories:

Spatial redundancy: Neighboring pixels are correlated so repeated data can be eliminated to reduce image size.

1. Temporal redundancy: For image representation, there is a need to reduce the number of the bits of the image data.

2. Spectral redundancy: Correlation between different color planes. To remove those redundancies, compression has to done for minimal storage and lower bandwidth transmission.

Image compression techniques are divided into two categories namely lossy and lossless compression techniques. Lossy image compression techniques remove the visually irrelevant information which inflicts losses on the reconstructed output image. Thus lossy image compression generates a reconstructed image with some degradation, whereas lossless preserves image information. The lossy compression

techniques achieve a high compression ratio. The removal of statistically irrelevant data is reversible or lossless compression. In which the image data can be completely retrieved from the compressed data. Thus the output of the lossless image compressor corresponds to the original image. Methods for lossy compression are:

1. Chroma subsampling
2. Transform coding
3. Fractal compression

The techniques involved in lossless image compression are:

1. Run-length encoding
2. Area image compression
3. DPCM
4. Entropy encoding
5. Adaptive dictionary algorithms
6. Deflation
7. Chain codes

The remaining of the paper is arranged with the following sections: Section 2 surveys the literature; Section 3 describes the proposed compression technique. The results and the comparison to the other methods are discussed in section 4, and section 5 concludes the work.

2. Overview of related and background works

In literature, various compression algorithms have been generated for color and grayscale images using SVD methods. The singular value decomposition packs signal energy into fewer coefficients [1,2]. It is a numerical technique which diagonalizes matrices in numerical analysis. It is also a reliable orthogonal matrix decompression method [3,4]. Awwal mohammed Rufai et al. [5] developed an image compression algorithm that boosts the performance of the WDR compression by cascading singular value decompression and wavelet difference reduction. Manoj Kumar et al. [6] developed a compression algorithm using SVD and Huffman encoding.

There is a great concern between the research communities in the implementation of image compression schemes that packs signal energy into fewer coefficients. Jin Wang et al. [7] analyzed an adaptive spatial post processing algorithm and used in block-based discrete cosine transform (BDCT) coded images. The method was comprised of three steps: a

thresholding, a model classification step, and a deblocking filtering step. Saiprasad Ravishankar [8] designed sparsifying transform such as wavelets and DCT that have been widely used in compression standards. He considered four different initializations. The first is the 64×64 2D DCT matrix. The second is obtained by inverting/transposing the left singular matrix. The third and fourth initializations were the identity matrix. K.M.M Prabhu et al. [9] invented a compression method, the extension of the warped discrete cosine transform known as the 3-D warped discrete cosine transform (3-D WDCT). The variation to DCT was known as warped discrete cosine transform (WDCT). The experimental results were shown that the 3-D WDCT performed better than 3-D DCT scheme.

Kaveh Ahmadi et al. [10] show the possibility of using the computational intelligence techniques Particle Swarm Optimization (PSO) for optimal thresholding in the 2-D discrete wavelet transform of an image. A set of optimal thresholds was obtained using the PSO algorithm. A variable length encoding scheme arithmetic coding was used to encode the results. The selection of optimal threshold value based on the PSO algorithm resulted with less entropy, lead to a gain in compression. Fouzi Douak et al. [11] proposed a lossy image compression algorithm based on the DCT transform and an adaptive block scanning. The efficiency of their scheme was demonstrated by results. They proposed such DCT based compression for color still images.

3. The proposed image compression technique

3.1. Singular Value Decomposition (SVD)

An image is a matrix whose elements are in numbers, and they are the intensity values of the corresponding pixels in an image. Hence singular value decomposition method is used to decompose the matrix into three matrices. Figure 1 shows the visualization of 2D singular value decomposition. The pink and the yellow directional arrows are the two canonical unit vectors. Initially, Z is in a disc shape, and at the next step, it changes to an ellipse. The SVD decomposes the matrix Z into three transformations:

1. An initial rotation Y^T
2. Scaling Σ along to the coordinate axes
3. Final rotation X

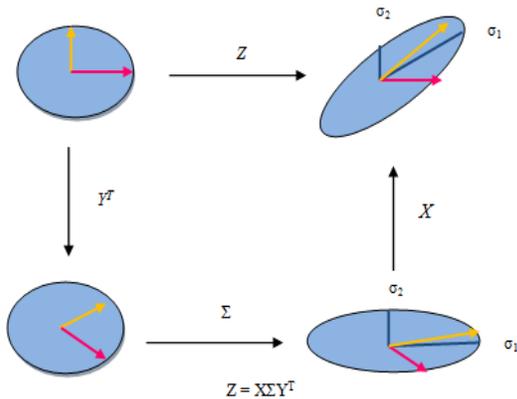σ_1 and σ_2 of the ellipse are the singular values (*Sv*) of Z.

Figure 1. Decomposition using SVD

Mathematically it can be formulated as follows: Any nonzero real matrix Z with rank r>0 can be decomposed into

$$Z = X \sum Y^T \qquad (1)$$

Where

X is an *m×r* orthonormal column matrix,

$$\Sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_r) \qquad (2)$$

Y^T is an orthonormal row matrix

Such factorization is called the singular value decomposition. This is related to the spectral theorem, such that if *A* is a symmetric matrix $A^T = A$, then

$$A = U \wedge U^T \qquad (3)$$

Where $\wedge$ is a diagonal matrix of eigenvalues, and *U* is an orthonormal matrix of eigenvectors.

To notice the relationship, consider:

$$Z^T Z = Y\Sigma^T X^T X\Sigma Y^T = Y\Sigma^2 Y^T \qquad (4)$$

$$ZZ^T = X\Sigma Y^T Y\Sigma^T X^T = X\Sigma^2 X^T \qquad (5)$$

These two are separately decomposed and hence σ_i are the positive square roots of the eigenvalues. The matrices are arranged, so that

$$\sigma_1 \geq \sigma_2 \geq \ldots \sigma_n \qquad (6)$$

Thus, an invertible matrix Z with *n×n* can be written as

$$Z = X\Sigma Y^T$$

$$= (x_1, x_2, \ldots x_n)\begin{pmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n \end{pmatrix}\begin{pmatrix} y_1^T \\ \vdots \\ y_n^T \end{pmatrix}$$

$$= x_1\sigma_1 y_1^T + x_2\sigma_2 y_2^T + \cdots + x_n\sigma_n y_n^T \qquad (7)$$

Since, $\sigma_1 \geq \sigma_2 \geq \ldots \sigma_n$, the terms $x_i\sigma_i y_i^T$ with small i contribute the sum, which contains the most relevant information about the image. These terms result in a lower image quality with limited storage requirements.

3.2. Butterfly Particle Swarm Optimization

In recent years, Artificial Intelligence based techniques have been deployed to solve complex problems for finding optimal solutions. BPSO is one of those techniques that gained popularity due to its advantage over other optimization techniques in many cases. This technique is an adaptation of Particle swarm optimization with two additional parameters: sensitivity and probability. PSO is based on flocking behavior of birds during their food search while BPSO relies on the intelligent characteristics and behavior of butterflies during their nectar search.

In particle based optimization techniques, the term particle is commonly used to refer fish, bird, ant, bee, etc., depending upon their consideration. In PSO, a flying bird at a position (location) with a velocity at any given time (displacement) in search of food is compared with a moving particle (singular value) at a position (current Sv) with a velocity at that given time (move to next Sv) in search of optimal solution (optimal Sv). Similarly, in BPSO, a flying butterfly is compared with a moving particle.

Let the size of butterfly swarm (the number of particles) be denoted by N. Let t denotes the iteration count and t_{max} denotes the maximum number of iterations. The position and velocity for a particle k are represented by vectors x_k and v_k respectively. Moreover, each particle k will maintain its personal best position at t denoted by $Lbest$. The overall best position achieved from the whole swarm at t is $Gbest$. The parameters of PSO so far discussed are included in BPSO along with two additional parameters: sensitivity S and probability P. Figure 2 illustrates the process of BPSO.
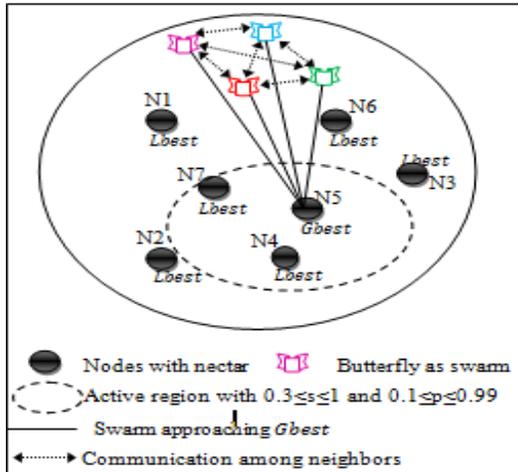
Figure 2. Butterfly Particle swarm optimization representation

Assume an initial search space with a butterfly swarm and nodes N_1 to N_6 representing flowers with nectar. The node is selected based on the presence of nectar. When the nectar source is present in a particular region, it is called active region. The region with no nectar is called inactive region. Hence, parameters S and P are used to decide flying direction towards active regions of the search space for each next iteration. By doing so, the optimal solution is obtained quickly and efficiently.

$$s(t) = \begin{cases} active\ , & 0.3 \leq s(t) \leq 1.0 \\ inactive, & otherwise \end{cases} \quad (8)$$

$$p(t) = \begin{cases} active\ , & 0.1 \leq p(t) \leq 0.99 \\ inactive, & otherwise \end{cases} \quad (9)$$

The equations of sensitivity and probability are given by,
$$s(t) = e^{-\left(\frac{t_{max}-t}{t_{max}}\right)} \quad (10)$$

Where: maximum number of iterations
    t : Current iteration
$$p(t) = \frac{Fit_{Gbest}}{\sum(Fit_{Lbest})} \quad (11)$$

Where: *Fitness* of local best solutions
    Fit_{Gbest} : *Fitness* of global best solution

The velocity and position update equations on iterations are given by,
$$v_i(t+1) = w(t).v_i(t) + s(t).(1-p(t)).r_1.c_1.(Lbest - x_i(t)) \\ + p(t).r_2.c_2.(Gbest - x_i(t)) \quad (12)$$

$$w(t) = \frac{t_{max}-t}{t_{max}} \quad (13)$$

Where r_1, r_2: uniformly distributed random numbers over [0,1]
    c_1, c_2: cognitive and social acceleration rates
    w : inertia weight

$$x_i(t+1) = x_i(t) + \alpha(t).v_i(t+1)\ ,\ 1 \leq i \leq N \quad (14)$$

$$\alpha(t) = rand.p(t), rand\ \epsilon\ [0,1] \quad (15)$$

Where: probability coefficient

The fitness function for selecting optimal singular value Sv is as follows:

$$Fitness = a \times entropy + \frac{b}{PSNR} \quad (16)$$

The parameters a and b are adjustable numbers provided to modify compression gain and distortion as per user requirements. For example, if compression is the goal then a should be increased and b should be decreased.

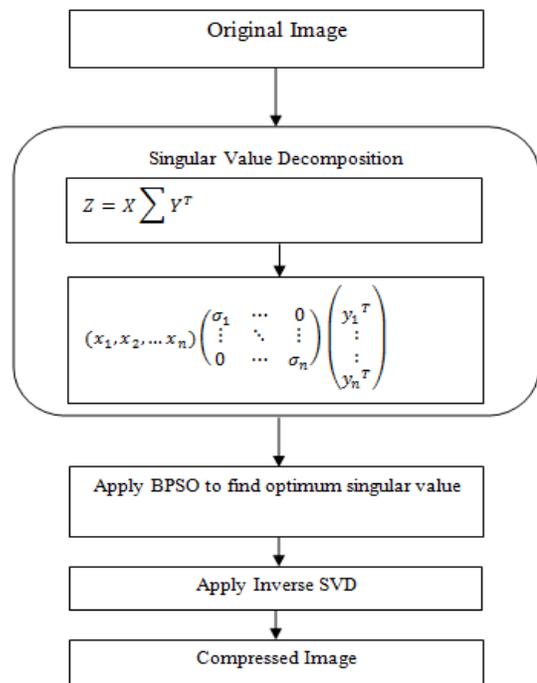### 3.3. Proposed SVD-BPSO image compression method

Figure 3. SVD-BPSO compression technique

Singular value decomposition (SVD) is used as an image compression technique, which compresses the image by decomposing image matrix into three matrices. The singular value Sv can be obtained from an eigen factorization. The quality of the compression method using SVDs depends on those singular values. Figure 3 is a schematic diagram of proposed SVD-BPSO compression technique. If more the singular value the quality will be high. However, unfortunately it stores images with more bits per pixel. The encoding time will also be more to finish the compression process. To remedy this problem, in the proposed SVD-BPSO method the butterfly particle optimization technique is incorporated to find the better quality reconstructed image by the entropy of the symbols.

The technique also reduces the encoding time, as it tries to find the singular value which results in high quality reconstructed compressed image. Implementation of BPSO over singular value decomposition based compression applies to the information security domain as it reduces the change of compromising over the network communication channel. The proposed compression technique is benefiting from cascading as this reduces the encoding time. SVD-BPSO is designed to increase the compression speed of the techniques. The proposed technique can also be used in cascading to minimize the time required during the encoding process. Butterfly particle swarm optimization (BPSO) is an artificial intelligent technique that has been evolved to find the problems in the optimal solution. In BPSO the flying butterfly is compared with a moving particle (Sv). Initially, the image is first divided into three matrices using singular value decomposition. Then BPSO is applied to find the optimum singular value. BPSO is an extended optimum technique of particle swarm optimization technique (PSO). The complete process of BPSO is already explained. In the proposed SVD-BPSO compression technique, the application of BPSO to find the optimum singular value leads to faster compression and reduces the complexity of generating a codebook.

4. Experimental results and discussions

4.1. The performance evaluation

4.1.1. Peak signal-to-noise ratio

The Peak Signal-to-noise ratio (PSNR) measures the difference in pixel value between two images, and

widely used to measure the quality of compressed or reconstructed images [12,13]. The definition of PSNR is:

$$PSNR = 10 * log\left(\frac{255^2}{MSE}\right) \qquad (17)$$

Mean Square Error:

The mean square error is the cumulative squared error between the original and the compressed image.

$$MSE = \frac{1}{MN}\sum_{i=0}^{M}\sum_{j=0}^{N}(I(i,j) - J(i,j))^2 \qquad (18)$$
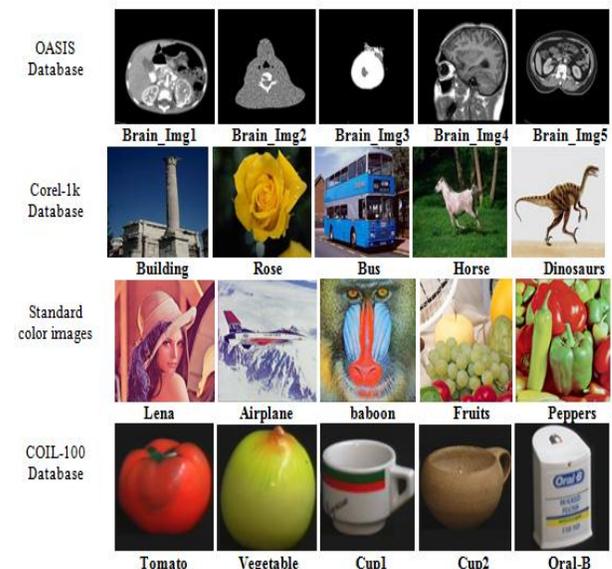
4.1.2 Compression Ratio

It is the ratio between uncompressed image size and the compressed image size.

$$Compression \quad Ratio = \frac{Uncompressed\ size}{Compressed\ size} \qquad (19)$$

4.1.3 Bits per Pixel

A number of bits used to represent single pixel turns bits per pixel.

$$bpp = \frac{Size\ of\ the\ compressed\ image\ in\ bits}{Total\ No.of\ pixels} \qquad (20)$$

Figure 4. Sample Input Images

Figure 4 shows some of the sample images contained in the databases to test the efficiency of the compression technique. The visual result of proposed SVD-BPSO compression technique is shown in Figure 5. Table 1 reveals the objection information of the SVD-BPSO compression algorithm. The compression performance significantly improves by reducing the execution time, increases the compression speed.

Figure 5. Subjective results of proposed SVD-BPSO image compression. (a) Sample Image (b) Reconstructed Image

Table 1: An Objective evaluation of the proposed SVD-BPSO compression for OASIS database

Images	PSNR (dB)	bpp	CR (%)	ET(sec)
Brain_Img1	35.2208	0.597	29.195	9.0598
Brain_Img2	32.3386	0.961	26.864	11.234
Brain_Img3	37.0604	0.896	28.972	9.6543
Brain_Img4	36.4085	0.747	29.873	9.7525
Brain_Img5	37.8472	0.992	27.321	9.2473

In the proposed SVD-BPSO method the best optimum solution is found by itself based on the fitness function. Table 2 shows the overall performance of the compression system. The quality reaches around 40dB PSNR for Tomato image through the proposed SVD-BPSO method and the encoding time is 8.6732 seconds which is also significantly low.

Table 2: Performance regarding PSNR, bpp, CR and ET for the proposed SVD-BPSO

Image Dataset	PSNR (dB)	bpp	CR (%)	ET(sec)
Standard Color Images				
Lena	29.8026	0.883	12.372	12.4566
Airplane	24.4695	0.773	12.267	11.5432
Baboon	24.3273	0.843	11.823	12.7156
Fruits	25.2695	0.682	17.412	12.4366
Peppers	29.6916	0.565	17.324	11.7321
Corel-1K				
Building	38.2217	0.521	9.23	23.5432
Rose	35.1723	0.743	7.23	15.6324

Bus	25.4321	0.623	6.43	17.1459
Horse	32.5731	0.823	8.74	11.3686
Dinosaurs	27.6916	0.672	9.47	10.43569
Coil-100				
Tomato	40.1641	0.4472	9.62	8.6732
Vegetable	38.7216	0.5736	8.762	8.56732
Cup 1	38.5672	0.4865	9.482	8.66932
Cup 2	38.7647	0.4256	9.761	8.77654
Oral-B	36.3467	0.4625	7.627	9.67432

For instance, Brain_Img1 achieved better compression speed than the previous algorithm. From Figure 6 (a)(b), in the SVD compression method, the execution or encoding time is more due to the singular values, whereas in the proposed SVD-BPSO the method itself finds the best optimum solution based on the fitness function The quantitative analysis proved that the method improved the compression performance in terms of ET and PSNR. To demonstrate the efficiency, the method is compared with the existing SVD compression technique. The average PSNR and the ET obtained by SVD are 31.956 dB and 104.677327 seconds respectively. The proposed SVD-BPSO outperforms SVD in both the quality and the compression speed in an average PSNR by 33.54 dB and 13.28968 seconds respectively.

(a)   (b)
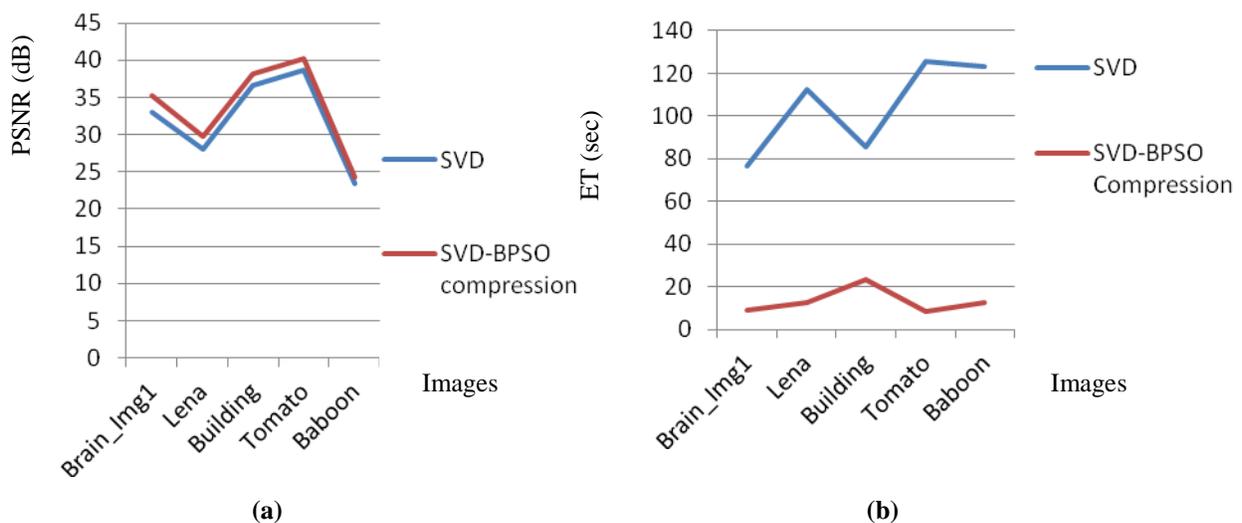
Figure 6. Quantitative Analysis (a) Comparison between SVD and the proposed SVD-BPSO algorithm by PSNR (b) Comparison between SVD and the proposed SVD-BPSO algorithm by encoding time (ET)

5. Conclusion

The proposed SVD-BPSO scheme is based on the optimization technique. It is shown that the performance of the proposed compression method is superior to SVD without optimization technique. The investigations have shown that the proposed method improves the compression speed. The integration of BPSO benefits the performance of the proposed method. A new method to improve the compression speed is presented. The optimization technique is combined with SVD based on the fitness function over the singular values. This is accomplished by determining the eigen values. The proposed SVD-BPSO method enhances the objective quality up to 1.59 dB in the average and decreases the encoding time up to 91.387692 seconds in average.

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