

# Performance Analysis of Existing Beam forming Methods for Various Antenna Elements and Interference Sources

Yashoda B.S<sup>1</sup> Dr. K.R. Nataraj<sup>2</sup>

*Ph.D. Research Scholar<sup>1</sup>*

*Jain University, Bangalore, India.*

Email ID: [Yashoda\\_bs@yahoo.com](mailto:Yashoda_bs@yahoo.com)

*Professor and Head<sup>2</sup>*

*Department ECE, SJBIT, Bangalore, India.*

Email ID: [nataraj.sjbit@gmail.com](mailto:nataraj.sjbit@gmail.com)

**Abstract:-** Antenna Arrays make use of techniques like Maximal Ratio Combining or Diversity is combining to achieve high Signal to Noise Ratio (SNR). The two kinds of the major algorithms used are Direction of Arrival (DOA) and Beam forming. This paper studies and performs the performance analysis of existing beam forming algorithms, namely Least Mean Square (LMS), Recursive Least Mean Square (RLS), Griffiths and Variable Step Size Griffiths (VSSG). The algorithms are simulated for various cases Low RF Sources and Single Interference, Large RF Source and Single Interference, Low RF Sources and Multiple Interference angles and finally in the case of Large RF sources and multiple interference angles.

**Key Words:** DOA, SNR, LMS, RLS

## I. Introduction

In today's world, the number of mobile users is increasing in an exponential format. With the limited electromagnetic spectrum, many users have to be served with the same QOS. Multiple antennas used as the base station increase the capacity in an efficient way by serving multiple users at the same time with the same frequency but different angle of orientation. The entire process is divided into transmission and reception dividends. The Reception is responsible for detecting the angles from which the sources are sending the electromagnetic waves, and the transmission part is in charge of sending the Radiation in the right direction and nullifies the jammer or interference directions.

## II. Background

The robust minimum variance [1] algorithm is a distortionless response (MVDR) beam former. The algorithm makes use of Kalman filter which reduces the computational cost.

In the paper[2] smart antenna concepts are described which increases the capacity by directing the beam in different directions on the same frequency using Least Mean Square (LMS), Recursive Least Mean Square Algorithm(RLS), Normalized Least Mean Square (NLMS) and Sample Matrix Inverse (SMI).

In the paper [3] the authors prove that the smart antenna algorithms increase the capacity and at the same time reduce the co-channel interference using LMS, RLS and SMI.

### III. Algorithms

Table: 1

| Notation                              | Meaning   |
|---------------------------------------|---|
| L                                     | Number of Antenna Elements  |
| $x(n)$                                | Received signal at the base station                                 |
| $d$                                   | Distance between antenna elements                                   |
| $a(\theta_0)$                         | Steering Vector for an angle $\theta_0$                             |
| A                                     | Manifold vector for multiple steering vectors                       |
| S                                     | Generated Signal Matrix at the Mobile Station of specific frequency |
| $\mu$                                 | Step Size   |
| M                                     | Number of Interference users  |
| $y(n)$                                | Total received signal at the base station                           |
| $e(n)$                                | Error signal  |
| $\theta_1, \theta_2, \dots, \theta_M$ | M interference angles   |
| $d(n)$                                | Received signal vector  |

#### 1. Least Mean Square (LMS) Algorithm

LMS [2] is the most accurate algorithm and then calculates the weight based on the computation of weight vector which reduces the mean square error. The weight vector is given by the following equation.

$$w(n+1) = w(n) + \mu e^*(n)x(n)$$

Where,  $\mu$  is the step size which can be in the range given by  $0 \leq \mu \leq \frac{2}{3tr(R_{xx})}$

$W(n)$  = weight for previous iteration.

$\mu$  = Step size

$e(n)$  = error signal which is the difference between received signal and actually generated signal

$x(n)$  = received signal at the base station.

The algorithm steps can be described as below

- Compute the  $L \times 1$  steering vector for the desired direction  $\theta_0$ .

$$a(\theta) = \begin{bmatrix} 1 \\ e^{i2\pi d \sin \theta} \\ \vdots \\ \vdots \\ e^{i2\pi d(L-1)\sin \theta} \end{bmatrix}$$

- Compute the  $L \times M$  array manifold vector corresponding to M interference source directions  $\theta_1, \theta_2, \dots, \theta_M$ .

$$A = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{i2\pi d \sin \theta_1} & e^{i2\pi d \sin \theta_2} & \dots & e^{i2\pi d \sin \theta_{M-1}} \\ \vdots & \vdots & \vdots & \vdots \\ e^{i2\pi d(L-1)\sin \theta_1} & e^{i2\pi d(L-1)\sin \theta_2} & \dots & e^{i2\pi d(L-1)\sin \theta_{M-1}} \end{bmatrix}$$

- Obtain signal samples 'S' by sampling continuous time signal of baseband frequency. (For simulation sine wave samples is considered).
- Compute the autocorrelation matrix which is the multiplication of the received signal with its hermitian transpose
- Compute the step size by computing the trace of a matrix
- Compute the following for all signal samples  $0 \leq n \leq N_s$ .

Where,  $N_s$  is the total number of signal sample.

#### 2. RLS Algorithm

RLS [2] algorithm is similar to that of steepest-descent is the weight vector is chosen to minimize the ensemble average of the error squares. To achieve reduced MSE the gain matrix is computed and used in the weight vector. The weight vector is computed using the following equations.

$$W(n) = W(n-1) + R^{-1}(n-1)x(n)[d(n) - c^H(n)W(n-1)]$$

$$R^{-1}(n) = \frac{1}{\alpha} R^{-1}(n-1) - \frac{1}{\alpha} \frac{R^{-1}(n-1)c(n)c^H(n)R^{-1}(n-1)}{1 + \frac{1}{\alpha} c^H(n)R^{-1}(n-1)c(n)}$$

$$R(n) = \frac{1}{\alpha} R(n-1) + c(n)c^H(n)$$

With initialization  $W(0) = 0$  and

$$R(0) = c(n) c^H(n) + v_I(n) v_I^H(n)$$

Where  $\alpha \cong 0.99$  is an exponential weight based forgetting factor?

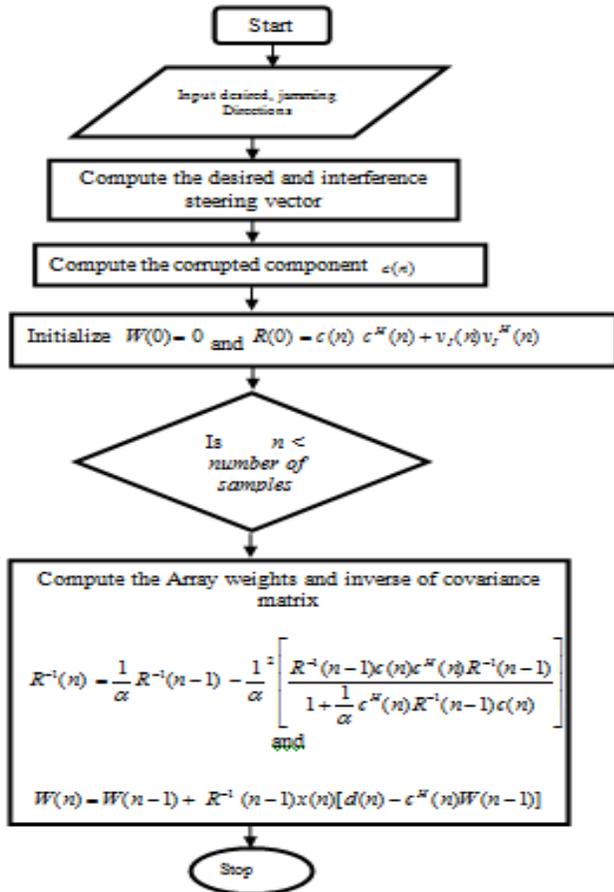


Fig1: RLS algorithm

### 3. Griffiths Algorithm

Griffiths' algorithm [4] utilizes certain a priori knowledge (when available) to create an efficient real-time adaptation process. The weight vector for the gifts algorithm is given by

$$\begin{aligned} w(n+1) &= w(n) + 2\mu e(n)x(n) \\ &= w(n) + 2\mu (d(n) - y(n))x(n) \\ &= w(n) + 2\mu r_{sx} - 2\mu y(n)x(n) \end{aligned}$$

Where  $\mu$  is the step size,  $r_{sx}$  is the cross-correlation and  $y(n)$  is the array output

- a. Compute  $L \times 1$  steering vector for the desired direction.

- b. Compute array manifold vector for jamming directions.
- c. Compute the cross-correlation matrix  $r_{sx}$ .
- d. Compute the following things for all signal samples  $0 \leq n \leq S_n$ .

Where  $S_n$  represents total number of signal samples

$$x(n) = S_\theta d(n) + \sum_{k=1}^M S(\theta_i) i(n) + n_k(n)$$

$$y(n) = W^H(n)x(n)$$

$$e(n) = s(n) - y(n)$$

$$W(n+1) = W(n) + 2\mu r_{sx} - 2\mu y(n)x(n)$$

### 4. VSSG Algorithm

The Variable Step Size Griffiths' [5] algorithm are a combination of the Variable Step Size LMS algorithm and the Griffiths' algorithm. It makes use of advantages of VSSLMS and Griffiths algorithm. The convergence is copied from VSSLMS and smoothness with Griffith's algorithm. The weight is computed using the following equation

$$W(n+1) = w(n) + \frac{\mu l}{\|x\|}$$

Where  $\|x\|$  is the norm of induced signal  $x(n)$ ?

The value of  $l$  is calculated by taking the difference between cross-correlation of reference signal with induced signal and cross-correlation between array output and induced signal given by

$$l = r - y^* x(n)$$

The upper bound for step size in VSSG is given by

$$\mu_{upper} = 0.07$$

The algorithm can be described using the following Steps.

- a. Compute  $L \times 1$  steering vector for the desired direction.
- b. Compute array manifold vector for jamming directions.
- c. Compute the following for all signal samples  $0 \leq n \leq S_n$ .

Where  $S_n$  represents a total number of signal samples.

$$x(n) = S_{\theta}d(n) + \sum_{k=1}^M (S(\theta_i)i(n) + n_k(n))$$

$$Y(n) = W^H(n) x(n)$$

$$e(n) = S(n) - y(n)$$

$$\mu(n+1) = \alpha\mu(n) + \gamma |e(n)|^2$$

$$\text{where } \mu(n+1) = \begin{cases} \mu_{\text{upper}} & \text{if } \mu(n+1) > \mu_{\text{upper}} \\ 0 & \text{if } \mu(n+1) < 0 \\ \mu(n+1) & \text{otherwise} \end{cases}$$

## IV. Results

### Set Up

**Table 2:**

| Parameter Name         | Parameter Value                           |
|------------------------|---|
| Type of Antenna        | Dipole                                    |
| Type of Array          | Uniform Linear Array                      |
| Variability            | $-90^{\circ} \leq \theta \leq 90^{\circ}$ |
| Antenna Separation     | $\frac{\lambda}{2}$                       |
| Noise                  | Random Noise                              |
| Frequency of Operation | 1MHz                                      |

### Case1: Low RF Elements and Single Interference User

| Parameter Name                  | Parameter Value |
|---------------------------------|-----------------|
| Number of Antenna Elements      | 8               |
| Desired Angle                   | 45              |
| Number of Interference Angle    | 1               |
| Direction of Interference Angle | 60              |

### Parameters for Simulation

#### 1. Radiation Pattern Polar Plots

This determines whether the algorithms are capable of forming the main beam in the look direction and then reduced or zero radiation at interference directions

#### 2. Mean Square Error

Mean Square is defined as the difference between actual signal and total received signal at the base station

$$MSE = |y(n) - s(n)|^2$$

Where,

$s(n)$  = generated signal

$y(n)$  = recieved signal at base station

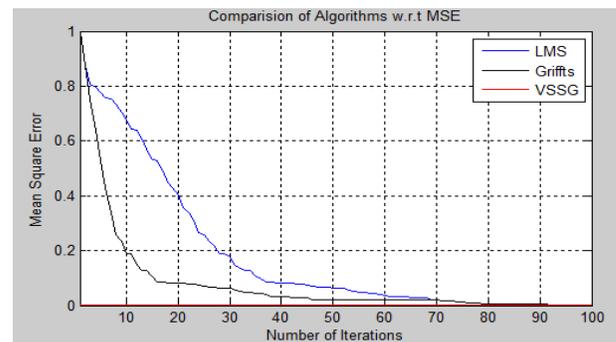
#### 3. Root Mean Square Error (RMSE)

If the beam forming algorithm is repeated for a range of angles, then the RMSE is the square root sum of all MSE values

$$RMSE = \sqrt{|y_{\theta}(n) - s_{\theta}(n)|^2}$$

$\theta$  = direction of desired user

$$10 \leq \theta + 5 \leq 70$$



**Fig2: MSE Comparison**

Fig2 shows the MSE comparison between LMS, Griffiths, and VSSG. VSSG has the lowest MSE as compared to Griffiths and LMS algorithm. The VSSG has the lowest MSE.

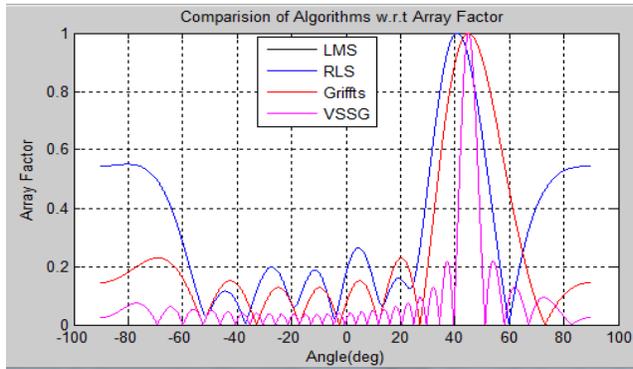


Fig3: Beam forming algorithm

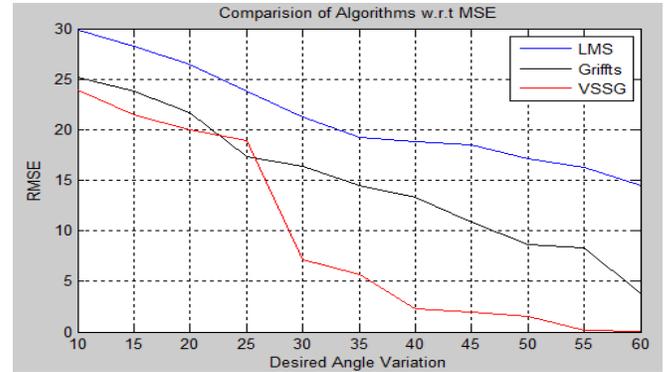


Fig4: RMSE Error

Fig 3 shows the radiation pattern for the beam forming algorithms; all the algorithms are capable of forming the main beam towards the desired user.

Fig 4 shows the RMSE plot, the RMSE of VSSG algorithm is the lowest as compared to Griffiths and LMS algorithms.

**Case2: Low RF Elements and Multiple Interference Users**

| Parameter Name                  | Parameter Value |
|---------------------------------|-----------------|
| Number of Antenna Elements      | 8               |
| Desired Angle                   | 45              |
| Number of Interference Angle    | 3               |
| Direction of Interference Angle | [10 30 60]      |

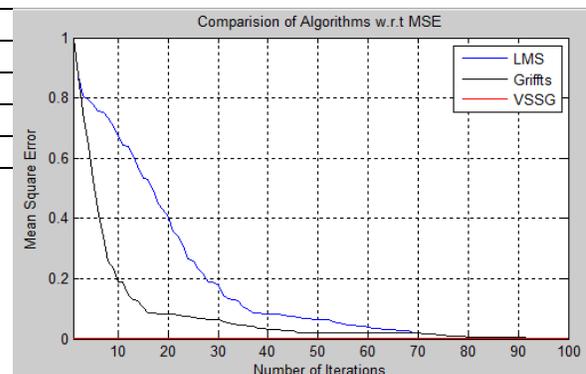


Fig5: MSE for Case2

Fig 5 shows the MSE comparison between LMS, Griffiths, and VSSG. VSSG has the lowest MSE as compared to Griffiths and LMS algorithm. The VSSG has the lowest MSE. For the case of low RF sources and multiple interference angles.

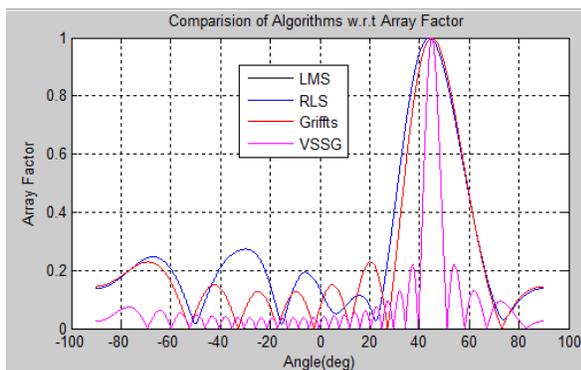


Fig6: Beam forming Algorithm Case2

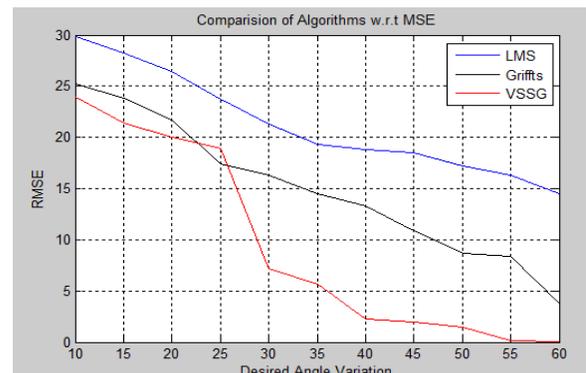


Fig7: RMSE Case2

Fig6 shows the beam forming algorithm, VSSG algorithm is the best as compared to other three algorithms, namely LMS, RLS, and Griffiths.

Fig7 shows the RMSE for VSSG is the best as compared to LMS and Griffiths.

**Case3: Large RF Elements and Single Interference User**

| Parameter Name                  | Parameter Value |
|---------------------------------|-----------------|
| Number of Antenna Elements      | 100             |
| Desired Angle                   | 30              |
| Number of Interference Angle    | 1               |
| Direction of Interference Angle | 45              |

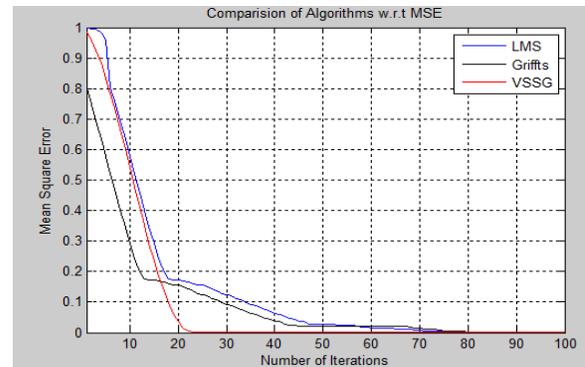


Fig8: Performance MSE Case3

Fig8 shows the MSE Performance for case 3; the VSSG converges for about 22 iterations as compared to LMS and Griffiths.

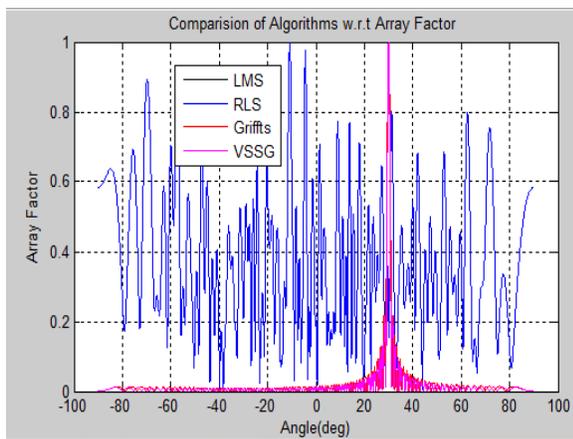


Fig9: Beam forming Algorithm Comparison Case3

Fig9 shows the radiation pattern of the RLS algorithm performance is worst as compared to the remaining algorithms and also the beam is becoming sharper for other algorithms as a number of antennas increases.

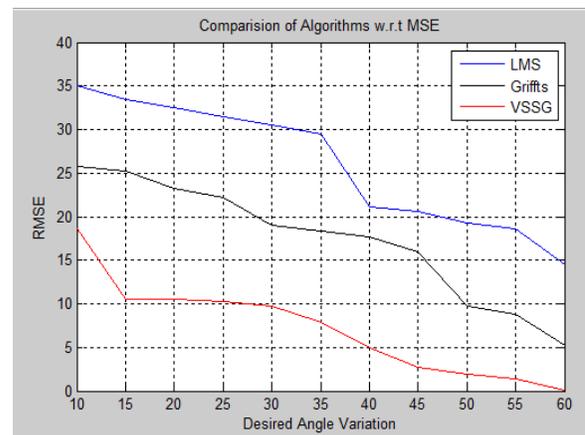
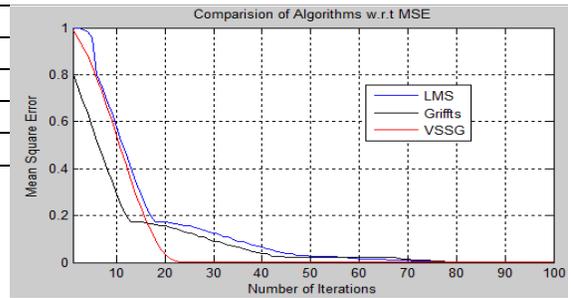


Fig10: RMSE Algorithm Results Case3

Fig10 shows the RMSE comparison of the VSSG is lowest, followed by Griffiths and last is LMS.

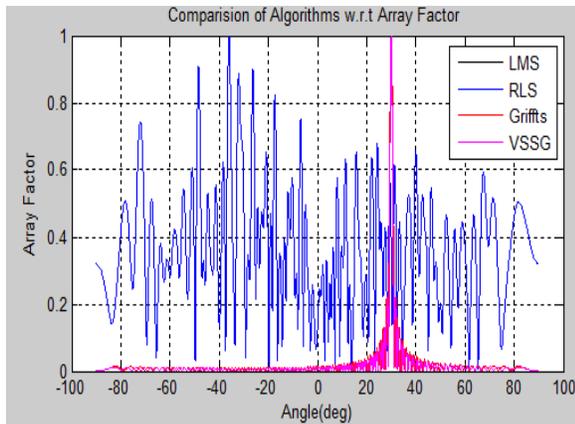
**Case4: Large RF Elements and Multiple Interference Users**

| Parameter Name                  | Parameter Value |
|---------------------------------|-----------------|
| Number of Antenna Elements      | 100             |
| Desired Angle                   | 30              |
| Number of Interference Angle    | 3               |
| Direction of Interference Angle | [10 45 60]      |

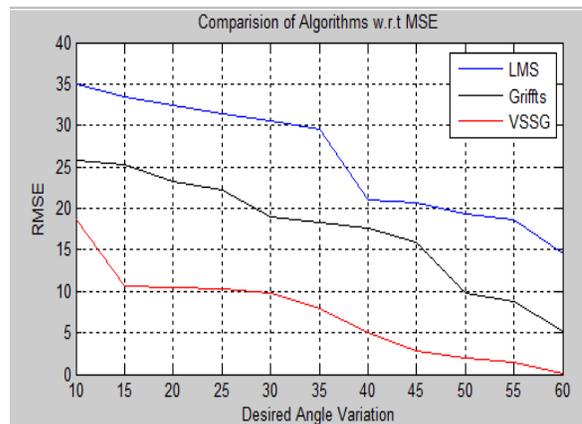


*Fig11: MSE Performance Analysis for Case4*

Fig11 shows the MSE Performance for the case 3 VSSG converges for about 22 iterations as compared to LMS and Griffiths. VSSG performs better than LMS and Griffiths.



*Fig12: Beam Forming Performance Analysis Case4*



*Fig13: Performance RMSE for Case4*

Fig12 shows the beam forming plots comparison as shown in the fig VSSG, LMS and Griffiths are better as compared to RLS Algorithm.

Fig13 shows the Performance of VSSG algorithm is best as compared to LMS and Griffiths for the case of the significant elements and multiple interference angles.

## V. Conclusion

From the simulation results for the various cases, the following conclusions can be drawn.

Case1: Low RF Sources and Single Interference Angle.

- a) The MSE of VSSG is the best, followed by Griffiths and last is the LMS.
- b) The Beam forming works in a nice manner for all the four algorithms, namely LMS, RLS, Griffiths, and VSSG.

- c) The RMSE of VSSG is the best as compared to all the remaining algorithms namely LMS and Griffiths.

Case2: Low RF Sources and Multiple Interference Angles

- a) The MSE of VSSG is the best, followed by Griffiths and last is the LMS.
- b) The Beam forming works in a nice manner for all the three algorithms ,namely LMS, RLS, Griffiths, and VSSG.

- c) The RMSE of VSSG is the best as compared to all the remaining algorithms namely LMS and Griffiths.

Case3: Large RF Sources and Single Interference Angles.

- a) The MSE of VSSG is the best, followed by Griffiths and last is the LMS.
- b) The Beam forming works in a nice manner for algorithms, namely LMS, Griffiths, and VSSG whereas for RLS it is worse.
- c) The RMSE of VSSG is the best as compared to all the remaining algorithms namely LMS and Griffiths.

Case4: Large RF Sources and Multiple Interference Angles.

- a) The MSE of VSSG is the best , followed by Griffiths and last is the LMS.
- b) The Beam forming works in a nice manner for algorithms , namely LMS, Griffiths, and VSSG whereas for RLS it is worse.
- c) The RMSE of VSSG is the best as compared to all the remaining algorithms namely LMS and Griffiths.

for the active control of noise in ducts. *J. Acoust. Soc. Am.* **95**(6) (1994)

[7] Kuo S.M., Morgan D.R.: Active noise control systems, algorithms and dsp implementations. Wiley, New York (1996) [Google Scholar](#)

[8] Kuo, S.M., Vijayan, D.: A secondary path modeling technique for active noise control. *IEEE Trans. Speech Audio* **5**(4) (1997).

[9] Kwong R.H., Johnston E.W.: A variable step-size LMS algorithm. *IEEE Trans. Signal Process.* **40**(7), 1633–1642 (1992).

## References

[1] "Robust adaptive beamforming based on the Kalman filter", A. El-Keyi ; T. Kirubarajan ; A.B. Gershman, *IEEE Transactions on Signal Processing* ( Volume: 53, Issue: 8, Aug. 2005.

[2] "Different adaptive beamforming algorithms for performance investigation of smart antenna system", Ashraf A. M. Khalaf ; Abdel-Rahman B. M. El-Daly ; Hesham F. A. Hamed, *Software, Telecommunications and Computer Networks (SoftCOM), 2016 24th International Conference* .

[3] *Comparative analysis of adaptive beamforming algorithm LMS, SMI and RLS for ULA smart antenna*", Dhaval N. Patel ; B. J. Makwana ; P. B. Parmar, *Communication and Signal Processing (ICCSP), 2016 International Conference on*

[4] Griffiths L.J.: A simple adaptive algorithm for real time processing of in antenna arrays. *Proc. IEEE* **57**, 1696–1704.

[5] S. V. Narasimhan, S. Veena, H. Loksha Variable step-size Griffiths' algorithm for improved performance of feedforward/feedback active noise control, *Signal, Image and Video Processing*, 2010, Volume 4, Number 3, Page 309.

[6] Kim, I.-S., Na, H.-S., Kim, K.-J., Park, Y.: Constraint filtered-X and filtered-U LMS algorithms